Monday 10/14 Lecture Notes

(From last class): If $a_1, \dots, a_M$ are vectors in $\mathbb{R}^n$ & $A = [\vec{a}_1, \dots, \vec{a}_M]$, $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}$

Then the following are equivalent

a) The set $\{\vec{a}_1, \dots, \vec{a}_M\}$ is linearly independent

b) The system $x_1 a_1 + \dots + x_M a_M = \vec{b}$ has at most one solution

c) $[\vec{a}_1, \dots, \vec{a}_M | \vec{b}]$ has at most one solution

d) $A\vec{x} = \vec{b}$ has at most one solution

Suppose $\{\vec{a}_1, \dots, \vec{a}_M\}$ is a linearly independent set of vectors in $\mathbb{R}^n$, b is in $\mathbb{R}^n$. Assume $c_1\vec{a}_1 + \dots + c_m\vec{a}_M = \vec{b}$ and $d_1\vec{a}_1 + \dots + d_M\vec{a}_M = \vec{b}$ thus $(c_1 - d_1)\vec{a}_1 + \dots + (c_m - d_m)\vec{a}_M = \vec{0}$. Since $\vec{a}_1, \dots, \vec{a}_M$ are linearly independent, $c_1 - d_1 = 0, \dots, c_m - d_m = 0$, or $c_1 = d_1, \dots, c_m = d_m$. Same solution.

Big Theorem:

$\mathcal{A}$ = set

A = matrix

Let $\mathcal{A} = \{\vec{a}_1, \dots, \vec{a}_n\}$ be n vectors in $\mathbb{R}^n$. (same # of vectors as vectors have entries) (rows = clmns) and $A = [\vec{a}_1, \dots, \vec{a}_n]$. Then the following are equivalent:

(a) $\mathcal{A}$ spans $\mathbb{R}^n$

(b) $\mathcal{A}$ is linearly independent

(c) $A\vec{x} = \vec{b}$ has a unique solution for all $\vec{b}$ in $\mathbb{R}^n$

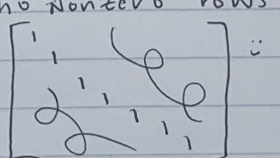
	$m < n$	$m = n$	$m > n$
Span	Span $\neq \mathbb{R}^n$	Span $= \mathbb{R}^n$ $\Updownarrow$	?
L.I.	?	Linearly Independent	Not linearly independent

m vectors in $\mathbb{R}^n$

3 vectors in $\mathbb{R}^3$ and they're LI, then they must span all of $\mathbb{R}^3$. 4 vectors in $\mathbb{R}^4$ and they span all of $\mathbb{R}^4$, they must be LI. 4 vectors in $\mathbb{R}^4$ that don't span all of $\mathbb{R}^4$ must be LD.

Visual Proof

If L.I., pivot in every column. If pivot in every column, no nonzero rows (# of columns = # of rows)



Matrices

Linear Transformations: A function T: from $\mathbb{R}^m \to \mathbb{R}^n$ is called a linear transformation if for all $\vec{u}, \vec{v}$, in $\mathbb{R}^n$, scalar c.

(a) $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$

(b) $T(c\vec{u}) = cT(\vec{u})$